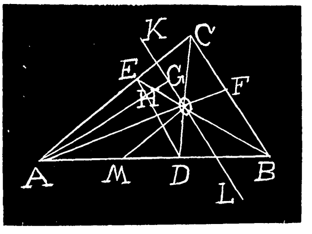
61. Proposed by WILLIAM E. HEAL, Member of the London Mathematical Society, and Treasurer of Grant County, Marion, Indiana.

Let the bisectors of the angles A, B, C of a triangle intersect in O and meet the sides opposite A, B, C in A', B', C'. Prove that the perpendiculars form O on the sides of the triangle $A'B'C'$ are $p_1=\frac{rR}{d_1}$, $p_2=\frac{rR}{d_2}$, $p_3=\frac{rR}{d_3}$ where r, R are the radii of the inscribed and circumscribed circles of the triangle ABC and d_1, d_2, d_3 are the distances of the center of the circumscribed circle from the centers of the escribed circles.

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Using trilinear coordinates, equation to CD is $\alpha-\beta=0$; to BE, $\alpha-\gamma=0$.

$$\therefore \left(\frac{2\triangle}{a+b}, \frac{2\triangle}{a+b}, 0\right), \left(\frac{2\triangle}{a+c}, 0, \frac{2\triangle}{a+c}\right),$$



are the coordinates of D, E.

$\therefore$ $\beta+\gamma-\alpha=0$, is the equation to DE.

The distance from O, (r, r, r) from this line is,

$$p_1=\frac{r}{\sqrt{3abc+2\cos C-2\cos A+2\cos B}}$$

$$=\frac{r}{\sqrt{\dfrac{3abc+a^2c+b^2c-c^3-ab^2-ac^2-b^3+a^2b+bc^2+a^3}{abc}}}$$

$$=\frac{r}{\sqrt{\dfrac{abc+(a+b+c)(a+b-c)(a-b+c)}{abc}}}=\frac{r}{\sqrt{\dfrac{abc+8s(s-b)(s-c)}{abc}}}$$

$$=\frac{r}{\sqrt{\dfrac{abc(s-a)+8\triangle^2}{abc(s-a)}}}=\frac{r}{\sqrt{\dfrac{\dfrac{abc}{4\triangle}+\dfrac{2\triangle}{s-a}}{\dfrac{abc}{4\triangle}}}}=\frac{r}{\sqrt{\dfrac{R+2r_1}{R}}}=\frac{rR}{\sqrt{R^2+2Rr_1}}=\frac{rR}{d_1}.$$

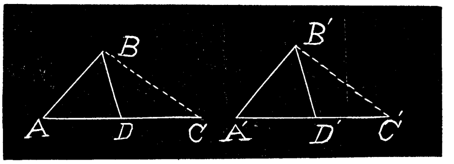
Similarly $p_2=\frac{rR}{d_2}$, $p_3=\frac{rR}{d_3}$.

62. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Prove that two triangles are equal if they have two sides and *the median of one of them* equal, each to each.

Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland, and CHAS. C. CROSS, Laytonsville, Maryland.

Let $AB=A'B'$, $AC=A'C'$, $BD=B'D'$. $\triangle ABD=\triangle A'B'D'$, because all the sides are equal, each to each.



Then $\triangle BDC=\triangle B'D'C'$, having two sides and included angle of one= two sides and included angle of the other.

$\therefore$ $\triangle ABC=\triangle A'B'C'$.

Also solved by *EDWARD R. ROBBINS*, *M. A. GRUBER*, and *G. B. M. ZERR*.

63. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O., University, Mississippi.

A rectangular hyperbola cannot be cut from a right circular cone *if the angle at its vertex is less than a right angle.*

Solution by the PROPOSER.

Let the base and the axis of the cone coincide with the xy-plane and the z-axis respectively. Then if c denote the altitude of the cone and ϕ the angle which any one of its elements makes with the base, its equation is

$$(x^2+y^2)\tan^2\phi=(z-c)^2.$$

The equation of a plane through the y-axis and inclined at an angle θ to the xy-plane is

$$z=x\tan\theta.$$

The projection on the xy-plane of the intersection of the two surfaces is

$$(x^2+y^2)\tan^2\phi=(x\tan\theta-c)^2=x^2\tan^2\theta-2cx\tan\theta+c^2.$$

This becomes, when referred to rectangular axes in the plane of the section, the origin and y-axis being unchanged, $(x^2\cos^2\theta+y^2)\tan^2\phi=x^2\sin^2\theta-2cx\sin\theta+c^2$, or $x^2(\cos^2\theta\tan^2\phi-\sin^2\theta)+y^2\tan^2\phi+2cx\sin\theta-c^2=0$, which represents a rectangular hyperbola if $\tan^2\phi+\cos^2\theta\tan^2\phi-\sin^2\theta=0$. From this equation,

$$\sin^2\theta=\frac{2\tan^2\phi}{\tan^2\phi+1}=2\sin^2\phi,\text{ and }\sin\theta=\pm\sqrt{2}\sin\phi.$$

Since $\sin\phi$ cannot be greater than $\frac{1}{\sqrt{2}}$, ϕ cannot exceed 45°. Hence the angle at the vertex of the cone cannot be less than 90°.

Other solutions of this problem will appear in next issue.

PROBLEMS.

67. Proposed by F. M. PRIEST, St. Louis, Mo.

Required: The length of a piece of carpet that is a yard wide with square ends, that can be placed diagonally in a room 40 feet long and 30 feet wide, the corners of the carpet just touching the walls of the room.